

Formulae to Solve Polynomial Equations

The Linear Formula

$$x = \frac{-b}{a}$$

The linear formula gives the solution of $ax + b = 0$ for real numbers a, b with $a \neq 0$.

The Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The quadratic formula gives the solutions of $ax^2 + bx + c = 0$ for real numbers a, b, c with $a \neq 0$.

The Cubic Formula

$$x = \frac{-b + \left(\frac{-1 + \sqrt{-3}}{2}\right)^n \sqrt[3]{4(-2b^3 + 9abc - 27a^2d + \sqrt{(-2b^3 + 9abc - 27a^2d)^2 - 4(b^3 - 3ac)^3})} + \left(\frac{-1 + \sqrt{-3}}{2}\right)^{n+1} \sqrt[3]{4(-2b^3 + 9abc - 27a^2d - \sqrt{(-2b^3 + 9abc - 27a^2d)^2 - 4(b^3 - 3ac)^3})}}{3a}$$

The cubic formula gives the solutions of $ax^3 + bx^2 + cx + d = 0$ for real numbers a, b, c, d with $a \neq 0$.

Directions: Take $n = 0, 1, 2$. Use real cube roots if possible, and principal roots otherwise.

The Quartic Formula

The quartic formula gives the solutions of $ax^4 + bx^3 + cx^2 + dx + e = 0$ for real numbers a, b, c, d, e with $a \neq 0$.

Directions: Choose all possibilities for the three \pm signs with the last two equivalent. Use real cube roots if possible, and principal roots otherwise.