

The Cubic Formula

$$x = \frac{-2b + \left(\frac{-1+\sqrt{-3}}{2}\right)^n \sqrt[3]{4(-2b^3 + 9abc - 27a^2d + \sqrt{(-2b^3 + 9abc - 27a^2d)^2 - 4(b^2 - 3ac)^3})} + \left(\frac{-1-\sqrt{-3}}{2}\right)^n \sqrt[3]{4(-2b^3 + 9abc - 27a^2d - \sqrt{(-2b^3 + 9abc - 27a^2d)^2 - 4(b^2 - 3ac)^3})}}{6a}$$

The cubic formula gives the solutions of $ax^3 + bx^2 + cx + d = 0$ for real numbers a, b, c, d with $a \neq 0$.

Directions: Take $n = 0, 1, 2$. Use real cube roots if possible, and principal roots otherwise.