

# The Quartic Formula

$$x = \frac{-b \pm \left( \sqrt{3(3b^2 - 8ac + 2a\sqrt{4(2c^3 - 9bcd + 27ad^2 + 27b^2e - 72ace) + \sqrt{(2c^3 - 9bcd + 27ad^2 + 27b^2e - 72ace)^2 - 4(c^2 - 3bd + 12ae)^3})} + 2a\sqrt{4(2c^3 - 9bcd + 27ad^2 + 27b^2e - 72ace) - \sqrt{(2c^3 - 9bcd + 27ad^2 + 27b^2e - 72ace)^2 - 4(c^2 - 3bd + 12ae)^3}} \right) \pm \sqrt{3(3b^2 - 8ac + 2a\sqrt{4(2c^3 - 9bcd + 27ad^2 + 27b^2e - 72ace) - \sqrt{(2c^3 - 9bcd + 27ad^2 + 27b^2e - 72ace)^2 - 4(c^2 - 3bd + 12ae)^3}}) + 2a\sqrt{4(2c^3 - 9bcd + 27ad^2 + 27b^2e - 72ace) - \sqrt{(2c^3 - 9bcd + 27ad^2 + 27b^2e - 72ace)^2 - 4(c^2 - 3bd + 12ae)^3}})}{4a} \pm \operatorname{sgn}\left(\left(\operatorname{sgn}(-b^3 + 4abc - 8a^2d) - \frac{1}{2}\right)\left(\operatorname{sgn}(\max(2c^3 - 9bcd + 27ad^2 + 27b^2e - 72ace)^2 - 4(c^2 - 3bd + 12ae)^3, \min(3b^3 - 8ac, 3b^4 + 16a^2c^2 + 16a^2bd - 16ab^2c - 64a^3e)) - \frac{1}{2}\right)\right) \sqrt{3(3b^2 - 8ac + 2a\sqrt{4(2c^3 - 9bcd + 27ad^2 + 27b^2e - 72ace) - \sqrt{(2c^3 - 9bcd + 27ad^2 + 27b^2e - 72ace)^2 - 4(c^2 - 3bd + 12ae)^3}}) + 2a\sqrt{4(2c^3 - 9bcd + 27ad^2 + 27b^2e - 72ace) - \sqrt{(2c^3 - 9bcd + 27ad^2 + 27b^2e - 72ace)^2 - 4(c^2 - 3bd + 12ae)^3}})}{4a}$$

The quartic formula gives the solutions of  $ax^4 + bx^3 + cx^2 + dx + e = 0$  for real numbers  $a, b, c, d, e$  with  $a \neq 0$ .

*Directions: Choose all possibilities for the three  $\pm$  signs with the last two equivalent. Use real cube roots if possible, and principal roots otherwise.*